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EXPERIMENTAL CONFIRMATION OF THERMO-ELASTIC EQUATIONS FOR SANDWICH PANEL DEFLECTION

I. K. EBCIOGLU
J. A. ROBERSON
J. R. SCHMERMUND
K. D. SHIMMIN

TECHNICAL DOCUMENTARY REPORT No. ASD-TDR-63-559

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AF MATERIALS LABORATORY
AERONAUTICAL SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

Project No. 7351, Task No. 735106

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FOREWORD

Report Bond

This report was prepared by the Creep and Dynamics Section, Strength and Dynamics Branch under Project No. 7351, "Metallic Materials", Task No. 735106, "Behavior of Metals". The work was administered under the direction of the Metals and Ceramics Division, AF Materials Laboratory, Aeronautical Systems Division, with Dr. I. K. Ebcioglu acting as project engineer.

The study presented began in October 1960 and was concluded in October 1962. The preliminary and experimental work was performed by the authors at ASD. The analytical work was performed at the University of Florida by Dr. Ebcioglu, who is now employed by that institution.

ABSTRACT

Deflection tests were performed on a sandwich panel made by Aeronca Manufacturing Corp. The panel was tested under a compressive edge load and with an arbitrary temperature distribution on the upper and lower faces. The center deflection of the panel was measured and compared with the calculated value. Good agreement has been found between experiment and theory.

This technical documentary report has been reviewed and is approved.

W. J. Trapp

W. J. TRAPP
Chief, Strength and Dynamics Branch
Metals and Ceramics Division
AF Materials Laboratory

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INTRODUCTION

The purpose of this investigation was to confirm the results of reference 1, which provides a means for predicting the deflection of a sandwich panel under conditions of an arbitrary temperature gradient, transverse load, and edge compression. These results are derived analytically, and provide a considerable simplification of sandwich panel theory. In the general case treated in reference 1, the sandwich panel has an orthotropic weak core, and the composition and the thickness of the faces is arbitrary. Conventional analytical treatment of such a panel under these conditions requires the solution of a set of five simultaneous differential equations.

The new theory makes use of the principles of virtual displacements and variational calculus, and by means of suitable transformation of the independent variables, two new differential equations are derived. These equations are suitable for predicting the deflection of a panel. Modern designs frequently include sandwich panel sections, and the design of the panels is extremely complicated; the simplification described above would be very desirable.

In the experiment designed to test the theory, a sandwich panel was loaded in transverse compression, and an arbitrary temperature gradient was maintained across and between its faces. All four edges of the panel were simply supported. The deflection perpendicular to the plane of the panel was recorded and compared to the theoretical estimate.

TEST EQUIPMENT AND TESTING PROCEDURE

In this experiment a stainless steel sandwich panel was mounted in a special buckling test apparatus which provides simple support to all four edges. A front to back temperature gradient was obtained by simultaneously heating and cooling opposite faces of the panel. When a stable temperature condition was reached, a compressive load was applied to the top and bottom edges and deflection was then measured at the center.

Loaded edges of the sandwich panel were supported by slotted disks which pivoted in ball bearings, while the unloaded edges were supported by slotted disks which were free to rotate in tubular side channels. Load was applied with a Tinius Olsen Universal Testing Machine, capacity 60,000 pounds. Emphasis was placed on deflection rather than on ultimate strength of the panel, since the purpose of this experiment was to check the theory.

The hot side of the panel was radiantly heated by quartz lamps; with a gold plated reflector being used to direct and concentrate the heat. The feedback type temperature control system used consisted of the following Research Incorporated equipment: Thermocouple Reference Junction Compensator, Recorder-Controller, and Thyratron Power Regulator. This system provided a means of maintaining a desired set temperature, at the control thermocouple, by automatic continuous regulation of current through the

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quartz lamps. The feedback or control thermocouple was located in the center of the hot side of the panel, as shown in figure 1. Details of the temperature control and monitoring equipment are shown in figure 2.

The cold side of the panel was cooled by a controlled blast of cold air. As shown in figure 3, compressed air was passed through a two-stage cooler, each stage consisting of coils of copper tubing immersed in a mixture of crushed dry ice and acetone. Cold air from the final cooling stage was directed at the center of the sandwich panel through nozzles oriented for optimum temperature distribution.

The temperature of each panel face was measured by eight Iron-Constantan thermocouples spotwelded to the various points as shown in figure 1. Temperature distribution over eight points was measured consecutively on the hot side, then on the cold side, by manual operation of the Hot-Cold Switch. Data were recorded by the use of a Brown Electronik Eight-Channel Continuous Balance Recording Potentiometer.

Deflection was measured by a parallel system consisting of a freely supported position-transferring rod perpendicular to and bearing against the panel center, on the cold side. This rod was mechanically linked to a differential transformer position transducer (Atcotran Microformer) and to a dial gage to double check the deflection.

Resultant electrical signals from the differential transformer and the load cell of the Universal Testing Machine were plotted in a conventional manner by the Atcotran Stress-Strain Recorder mounted on the Universal Testing Machine. Further details of the instrumentation set up may be seen in figure 3.

It was found that the most satisfactory and stable temperature distribution was obtained when the heating and cooling sources were operated simultaneously. The desired temperature distribution could be maintained for a maximum time of approximately five minutes, after which icing in the cold air supply system restricted flow.

NUMERICAL CALCULATIONS AND DISCUSSION

The experiments were conducted on a sandwich panel made of 17-7PH stainless steel face sheets with an hexagonal core. The dimensions and physical properties of the panel are given in table 1. The transverse shear moduli of the panel are calculated from equations (13) and (14) of reference 2.

$$\frac{\bar{G}_y}{G} \cdot \frac{c}{t_f} = \frac{1 + \cos^2\theta}{(1+\cos\theta)\sin\theta} \tag{1}$$

$$\frac{G_x}{G} \cdot \frac{c}{t_f} = \frac{\sin\theta}{1 + \cos\theta} \tag{2}$$

The definitions of C, t_f and θ are shown in figure 4. G is the shear modulus of the core faces. From these equations transverse shear moduli are found to be

$$\bar{G}_y = 115{,}000 \text{ psi}$$

$$\bar{G}_x = 43{,}125 \text{ psi}$$

The theoretical deflection for a temperature gradient of cosine form can be obtained from reference 1 as:

$$w_2 = \hat{w}_2 \frac{8}{\pi^2}(1+\beta^2) \frac{\left[(1-\mu)(1+\bar{g}\beta^2) r_e + 2\bar{g}\right] \cos\frac{\pi}{2}\xi \cos\frac{\pi}{2}\eta}{(1+\bar{g}\beta^2)\left[(1-\mu)(1+\beta^2)^2 r_e + 2(\bar{g}+\beta^2)\right] - 2\beta^2(\bar{g}-1)^2 - k_{ie}\left\{\left[(1-\mu)(1+\beta^2)^2 r_e + 2(\bar{g}+\beta^2)\right] r_e + \left[(1-\mu)(1+\bar{g}\beta^2) r_e + 2\bar{g}\right]\right\}} \tag{3}$$

where the values of the parameters appearing in this equation are given in table 2. Using these parameters the deflection of the sandwich panel can be obtained in the following form:

$$w_2 = \frac{189 \times 10^{-6}(T_1 - T_2)\cos\frac{\pi}{2}\xi \cos\frac{\pi}{2}\eta}{1 - 0.275\, k_{ie}} \tag{4}$$

where $T_1 - T_2$ is the temperature difference of the faces which will be calculated subsequently.

The measured temperatures of the hot and cold side of the sandwich panel are given in figures 5 and 6 respectively. It can be seen from the figures that the shape of the temperature distribution of the hot side is cylindrical while that of the cold side is paraboloidal. The temperature distribution of the neutral plane of the sandwich panel can easily be obtained from figures 5 and 6 by taking the arithmetical average of the temperatures of the hot and cold sides. The result of this calculation is illustrated in figure 7, and the equal temperature (isothermal) curves are drawn. Equation (3) is obtained with the assumption that the temperature distribution of the neutral plane, figure 7, is uniform. It can be seen from figure 7, except for the small portion near the vertical edge, this variation is less than $\pm 3°$F. Therefore, we reasonably expect that equation (3) can satisfactorily be used for this case. Finally, the spatial distribution of the temperature gradient is given in figure 8, which is obtained from figures 5 and 6 by taking the difference of the temperatures of the hot and cold sides. Using the temperature gradient at the center of the plane, $T_1 - T_2 = 52°$F and putting $k_{ie} = 0$ we obtain, from equation (4),

$$w_2 = 0.0098 \text{ in.}$$

It can be seen from figure 9 that in the above calculation we assume a temperature gradient of cosine shape having an amplitude equal to the maximum of the measured temperature gradient. This deflection will be smaller than the actual deflection due to the shaded area which is not considered in the aforementioned calculation. In order to find an upper limit we plot another cosine curve in such a way that the area under this curve and the curve of the actual temperature gradient distribution are equal. In this way the temperature gradient distribution at the central part of the sandwich panel was increased. Therefore, it is reasonable to expect that this temperature distribution will yield a theoretical deflection larger than the deflection which was observed. For this case, $T_1 - T_2 = 67°F$, and the corresponding deflection can be calculated from equation (4) as,

$$w_2 = 0.0126 \text{ in.}$$

From the above argument the actual deflection must lie between these lower and upper limits, that is,

$$0.0098 < w_2 < 0.0126$$

The measured deflection at the center of the sandwich panel was 0.013 in. If $w_2 = 0.011$ in. is taken for the calculated value, the relative error is found to be 15 percent which shows good agreement between theory and experiment. Better agreement would be found if the exact shape of the temperature distribution would be considered. These calculations require the use of computers.

In order to find the deflections of the sandwich panel at any point under the above temperature gradient as well as axial loading the following formula may be used:

$$w_2 = \frac{0.011}{1 - 0.275 k_{ie}} \cos \frac{\pi}{12} x \cos \frac{\pi}{12} y$$

which is obtained from equation (4). As an example, if

$$P_y = 1000 \frac{\text{lb.}}{\text{in.}}, \quad k_{ie} = \frac{P_y}{P_e} = 0.6, \text{ then}$$

$$w_2 = 0.0132 \text{ in.}$$

REFERENCES

1. Ebcioglu, I. K., "Bending of Sandwich Panels Under Thermal and Mechanical Loads." Submitted for review to the 8th Midwestern Mechanics Conference, Case Institute of Technology, Cleveland, Ohio.

2. Chang, C. C., and Ebcioglu, I. K., "Effect of Cell Geometry on the Shear Modulus and on Density of Sandwich Panel Cores." Journal of Basic Engineering, pp. 513-518, December, 1961.

APPENDIX

TABLE I

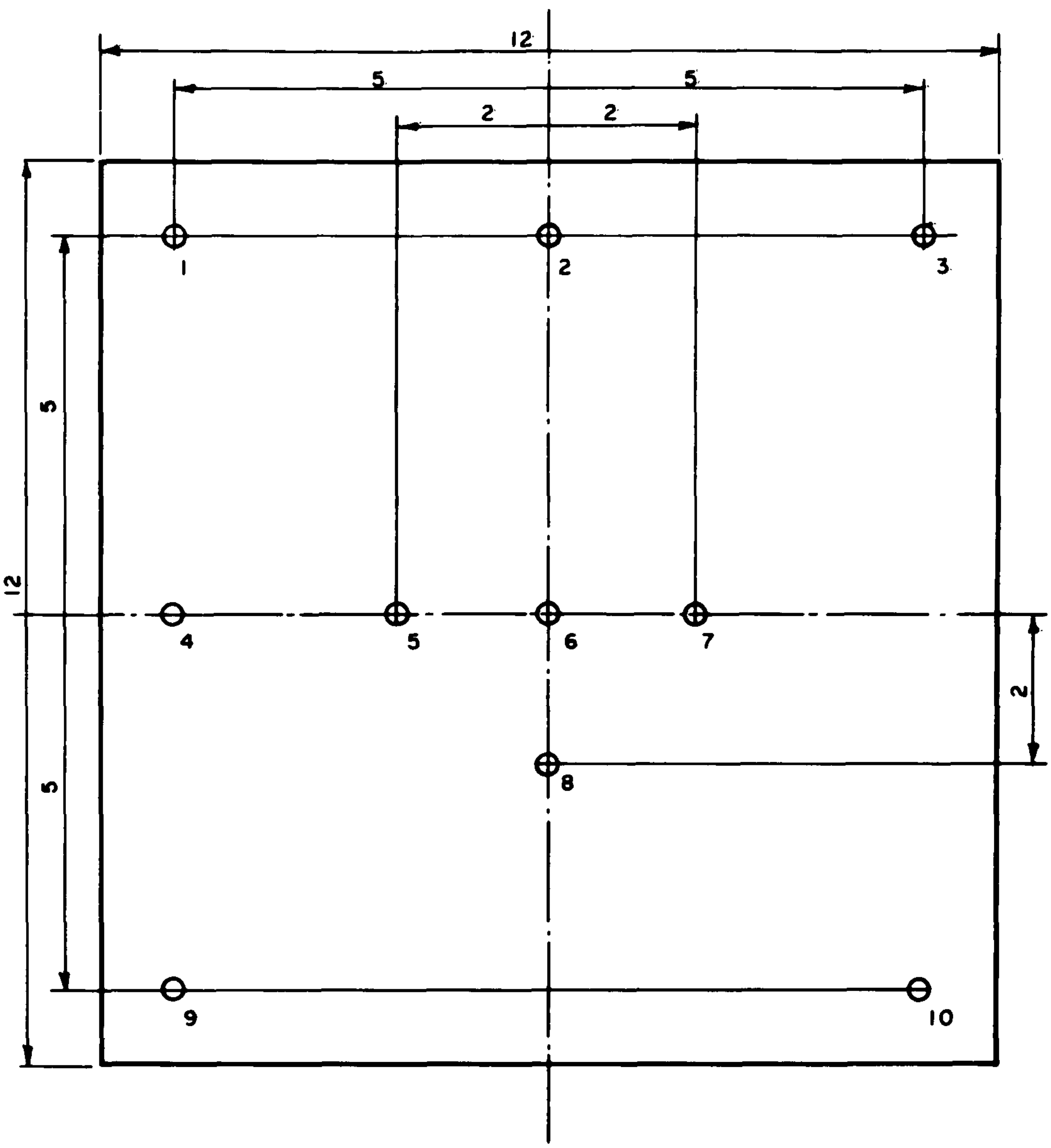
SPECIFICATIONS FOR AERONCA SANDWICH PANEL

t = FACING THICKNESS:	0.010"
t_f = CORE FACE THICKNESS:	0.0015"
c = CELL SIZE:	0.25"
$\bar{t}$ = CORE THICKNESS:	0.395"
a = b = PANEL HALF-LENGTHS:	6.0"
μ = POISSON'S RATIO:	0.3
E = YOUNG'S MODULUS:	30×10^{6} psi
G = SHEAR MODULUS:	11.5×10^{6} psi
α = COEFFICIENT OF THERMAL EXPANSION:	7.34×10^{-6} in./in.°F

NOTE: SUPERSCRIPT BAR VALUES REFER TO CORE PROPERTIES

TABLE 2

CALCULATIONS FOR AERONCA SANDWICH PANEL
$P_e = \frac{(\bar{t}+t)^2}{1-\mu^2}\left(\frac{\pi}{2b}\right)^2 \frac{Et}{2} = 1{,}671$ lb/in.
$\bar{H}_y = \bar{t}\,\bar{G} = 43{,}125$ lb/in.
$\hat{t} = \frac{\bar{t}+t}{\bar{t}} = 1.023$
$r_e = \frac{P_e}{\hat{t}^2 \bar{H}_y} = 0.037$
$\alpha = 7.34 \times 10^{-6}$ in./in. F°
$a = b = 6$ in.
$\bar{\alpha} = \frac{\bar{t}}{a} = 0.0602$
$\hat{w}_z = \frac{\alpha b(1+\mu)}{2\hat{t}\,\bar{\alpha}}(T_1 - T_2) = 4.67 \times 10^{-4}(T_1 - T_2)$ in.
$\bar{g} = \frac{\bar{G}_x}{\bar{G}_y} = 0.6$
$\beta = \frac{a}{b} = 1$
$\xi = \frac{x}{a}$
$\eta = \frac{y}{b}$
$k_{ie} = \frac{P_y}{P_e}$



NOTE: FRONT AND BACK SIDES ARE IDENTICAL. NO. 6 IS THE CONTROL POINT; NO. 10 NOT USED.

Figure 1. Thermocouple Location

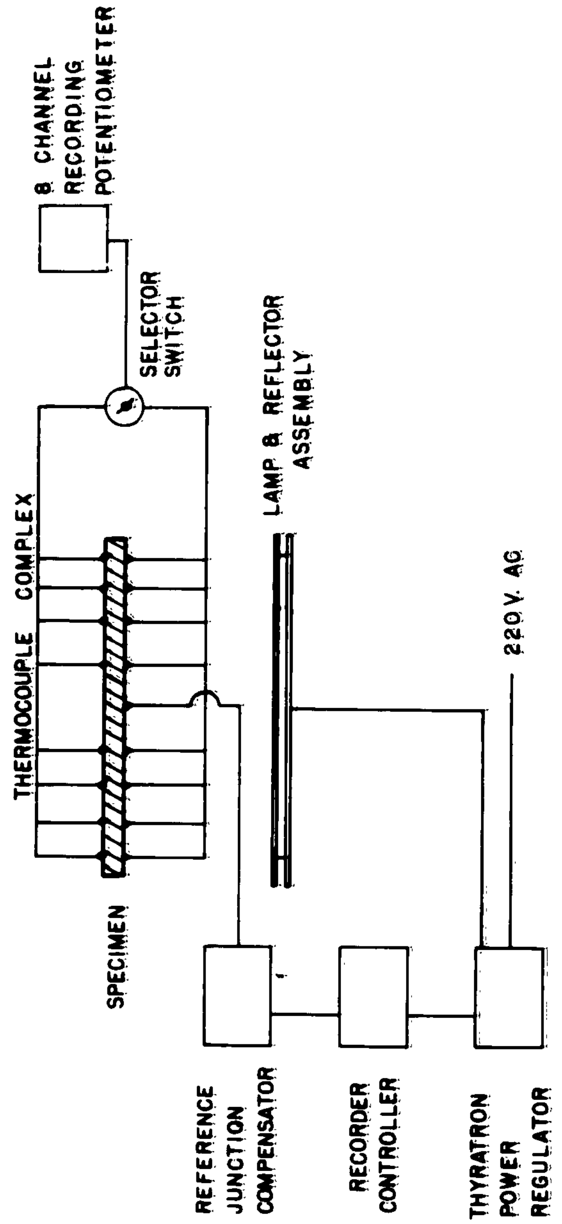


Figure 2. Instrumentation for Monitoring and Controlling Temperature

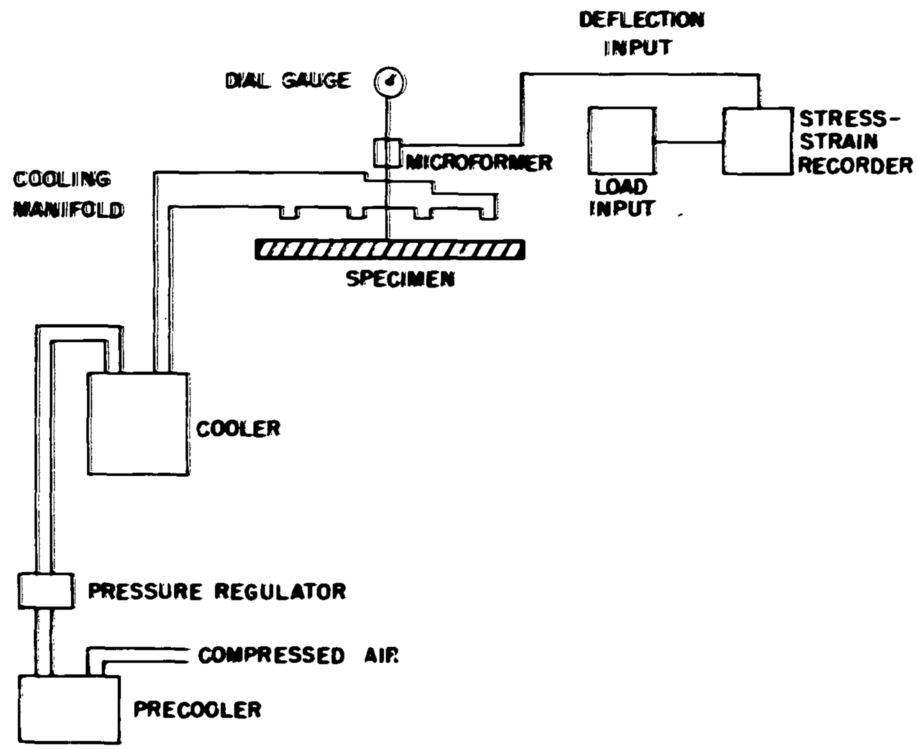


Figure 3. Deflection Measuring and Specimen Cooling Apparatus

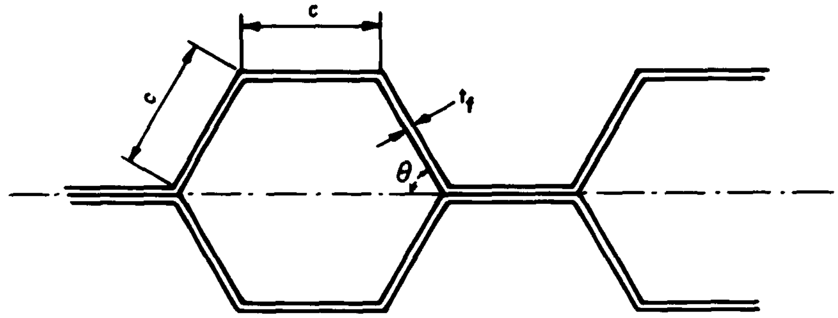


Figure 4. Configuration of Core Cell

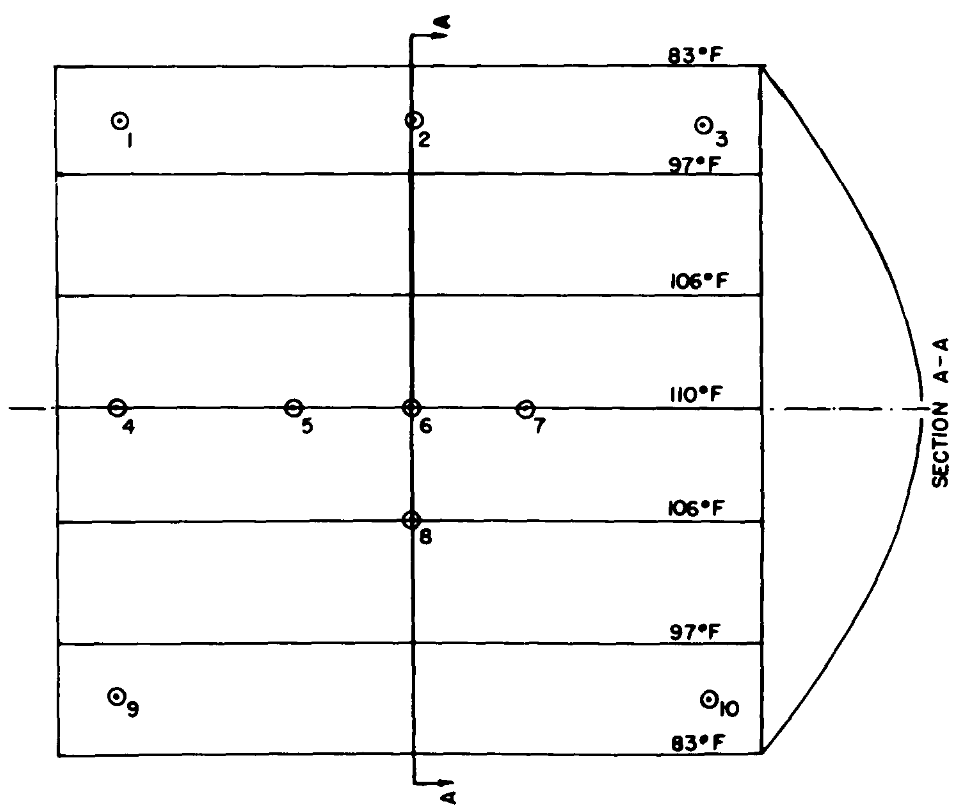


Figure 5. Temperature Distribution of the Hot Side of the Sandwich Panel

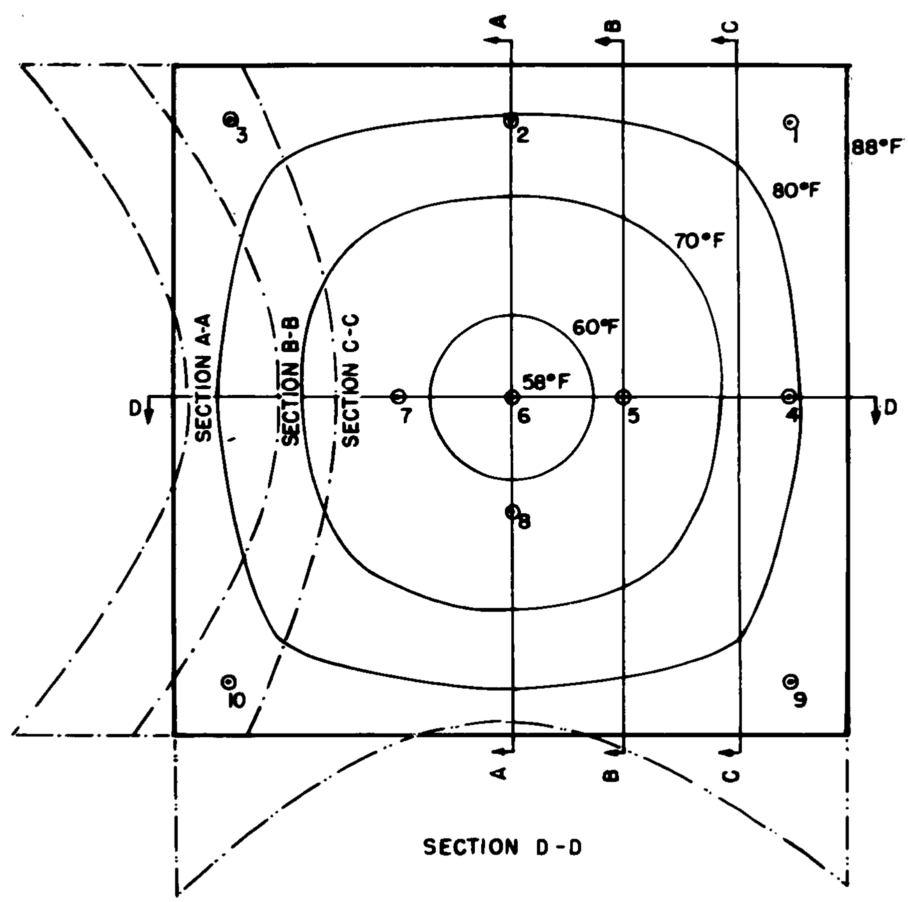


Figure 6. Isothermal Curves and Temperature Profiles on the Cooled Face

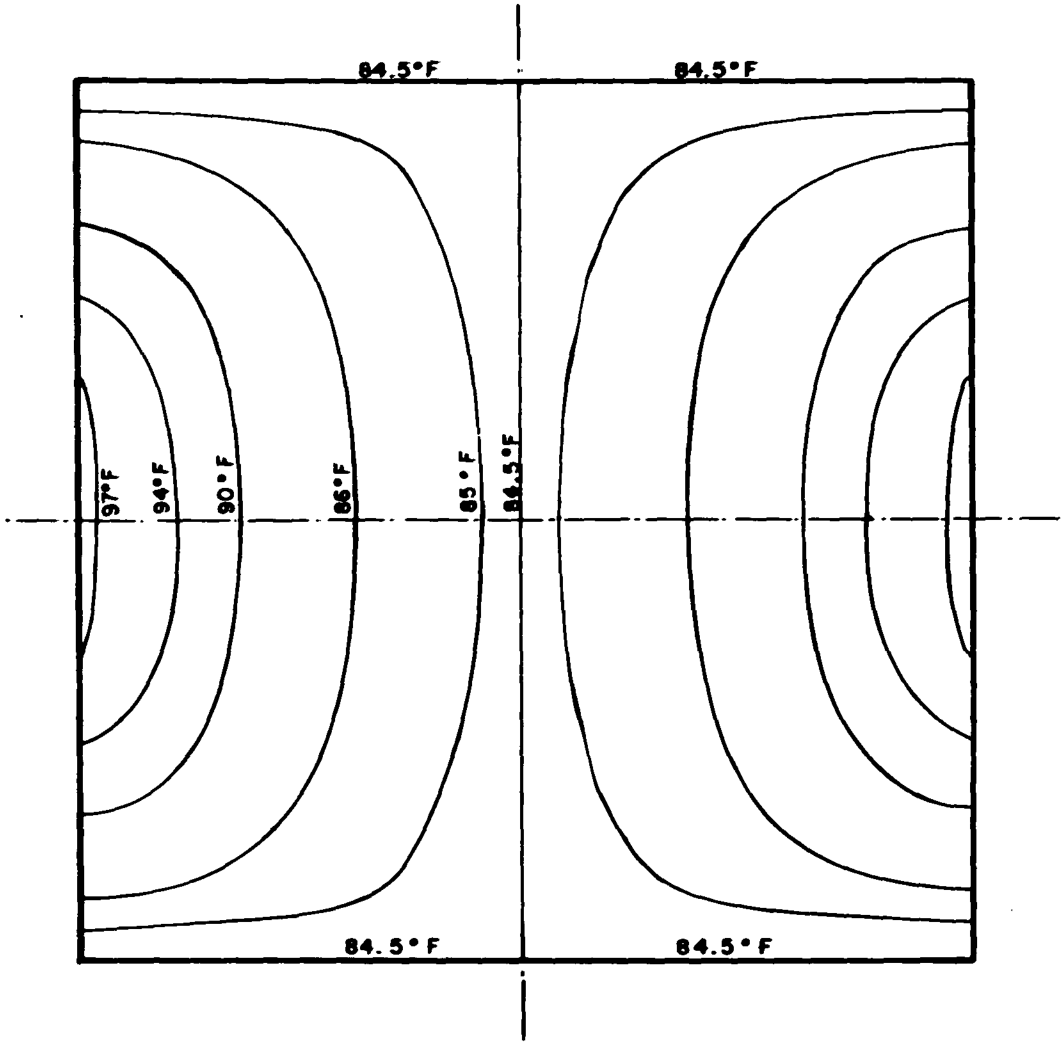
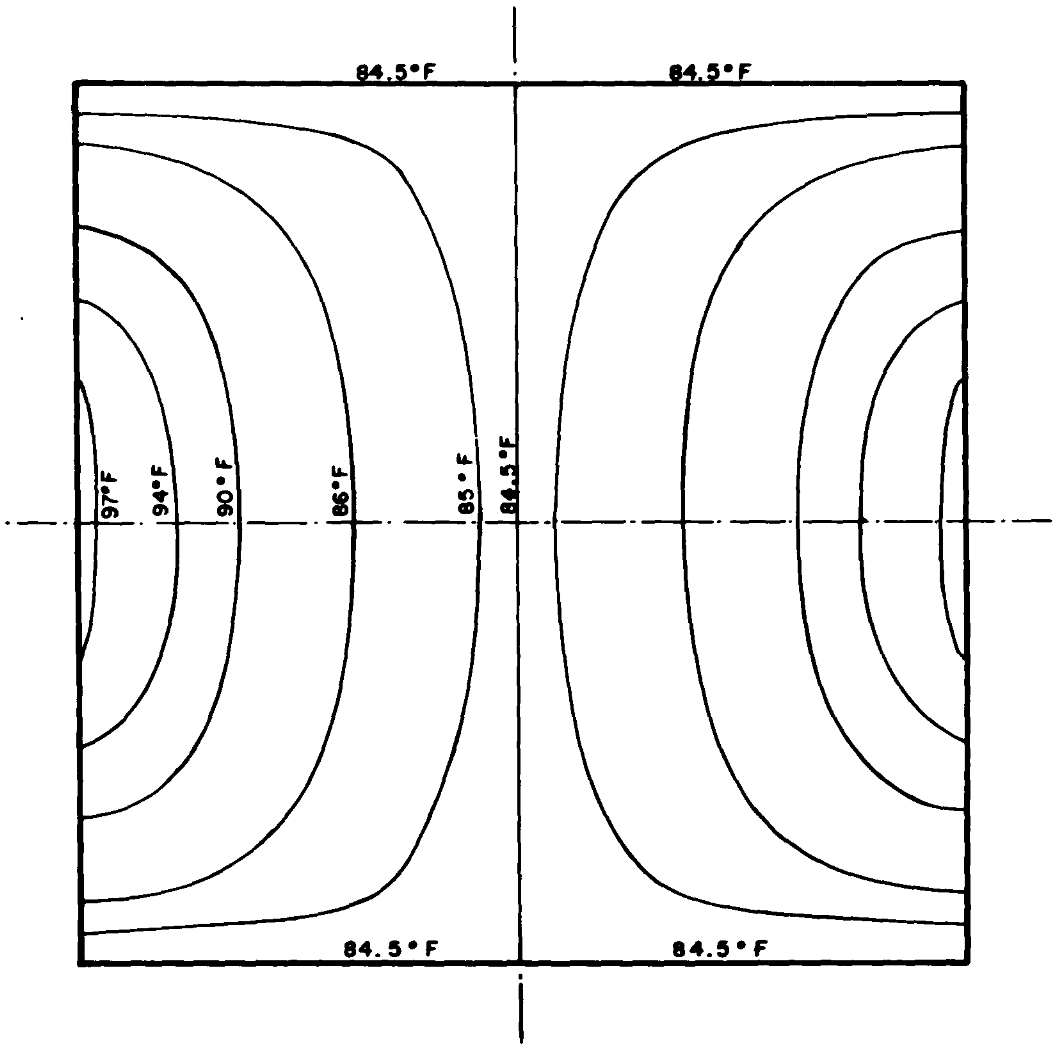


Figure 7. Isothermal Curves at the Neutral Plane



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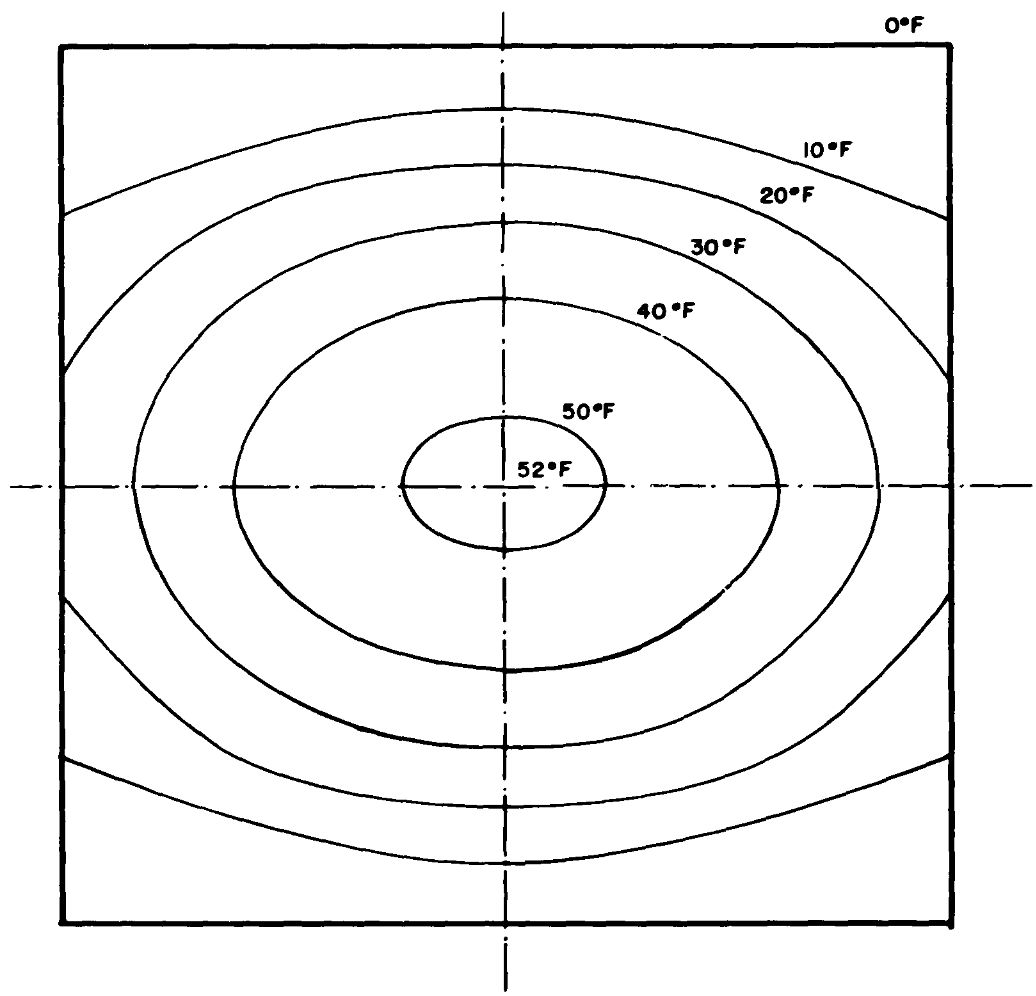

Figure 8. Distribution of Temperature Differentials ($\hat{t}\,\bar{t}$ gradient)

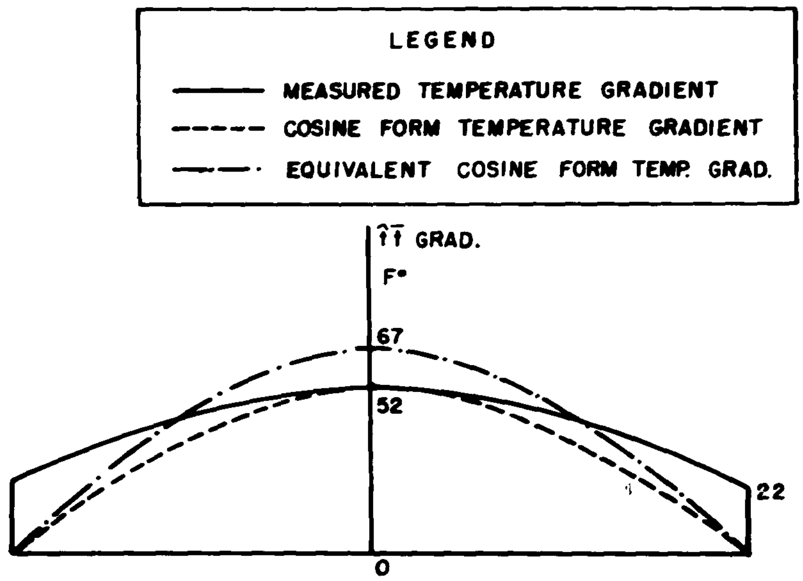

Figure 9. Variation of the Temperature Differentials ($\hat{t}\ \bar{t}$ gradient) at the Middle Section of the Sandwich Panel

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